Math 5A Quiz 5 : 2.3ii- 2.5

Show all work neatly with clear presentations and correct notation. 20 points

1) Find the derivative and simplify. (remember you can simplify f before differentiation if it helps) (3 points each)

a) $f(x)=\frac{2x^3}{5}\cos x = \frac{2}{5}x^3\cos x$

$f'(x)=\frac{2}{5}\frac{d}{dx}(x^3\cos x)$ factor out constant

$=\frac{2}{5}\left[\frac{d}{dx}(x^3)\cos x + x^3\frac{d}{dx}(\cos x)\right]$ product

$=\frac{2}{5}(3x^2\cos x - x^3\sin x)$

$=\frac{6}{5}x^2\cos x - \frac{2}{5}x^3\sin x$

Not required to show all these steps, but it can be helpful while learning

b) $f(x)=\frac{\tan x\cos x}{2x^2+1} = \frac{\frac{\sin x}{\cos x}\cos x}{2x^2+1}$ simplify f first

$f(x)=\frac{\sin x}{2x^2+1}$

$f'(x)=\frac{(2x^2+1)\frac{d}{dx}(\sin x)-\sin x\frac{d}{dx}(2x^2+1)}{(2x^2+1)^2}$

$f'(x)=\frac{(2x^2+1)\cos x-\sin x(4x)}{(2x^2+1)^2}$

$=\frac{2x^2\cos x+\cos x-4x\sin x}{(2x^2+1)^2}$

c) $g(t)=\sqrt{9-t^2} = (9-t^2)^{1/2}$

$g'(t)=\frac{1}{2}(9-t^2)^{-1/2}\frac{d}{dt}(9-t^2)$

$=\frac{1}{2}\frac{1}{\sqrt{9-t^2}}(-2t)$

$g'(t)=\frac{-t}{\sqrt{9-t^2}}$

Make sure to
- label derivatives
- simplify
  - no complex fractions
  - no negative exponents
  - combine fractions

2). Find the derivative: $f(x)=\dfrac{3x^2}{\sqrt[3]{2x+5}} = 3x^2(2x+5)^{-1/3}$ (4 points)

(or use quotient rule)

product

$f'(x) = \frac{d}{dx}(3x^2)(2x+5)^{-1/3} + 3x^2\frac{d}{dx}(2x+5)^{-1/3}$

chain

$= 6x(2x+5)^{-1/3} + 3x^2 \cdot \frac{-1}{3}(2x+5)^{-4/3}\frac{d}{dx}(2x+5)$

$= 6x(2x+5)^{-1/3} - 2x^2(2x+5)^{-4/3}$

factor

$= (2x+5)^{-4/3}(6x(2x+5) - 2x^2)$

$= \dfrac{12x^2+30x-2x^2}{(2x+5)^{4/3}} = \dfrac{10x^2+30x}{(2x+5)^{4/3}}$

This was the example on video 3 of 2.3

3) Find the equation of the tangent line(s) to $f(x)=x^3$ that contains the point $\boxed{(2,0)}$. (4 points)

Attach a computer generated graph which clearly validates your results. (3 points)

Point of tangency $(a, f(a)) = (a, a^3)$ slope $= f'(a) = 3a^2$

Line: $y - a^3 = 3a^2(x-a)$

$(2,0)$ is a point on the line $\Rightarrow$

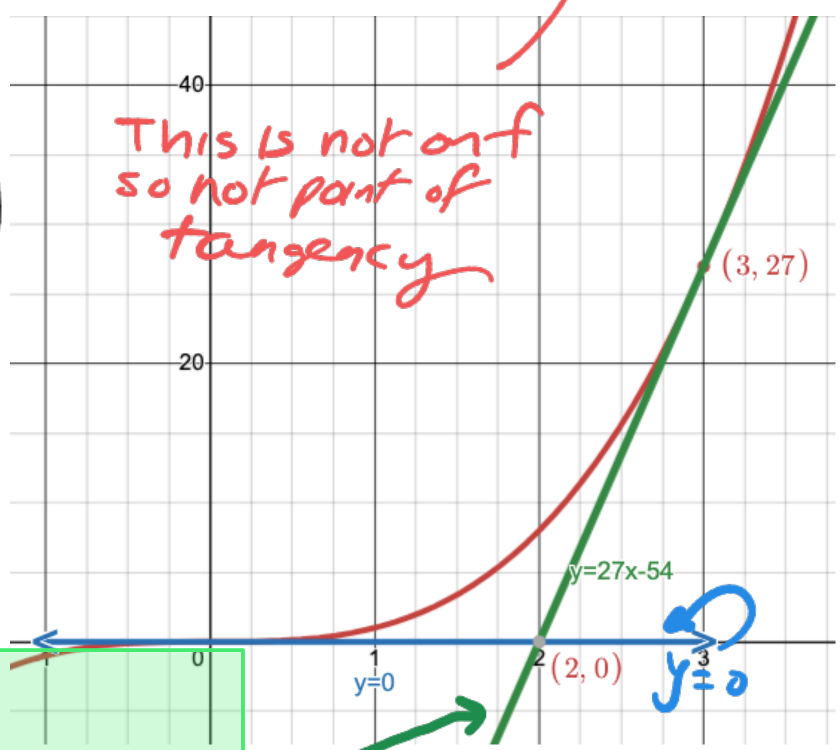
$0 - a^3 = 3a^2(2-a)$

$-a^3 = 6a^2 - 3a^3$

$2a^3 - 6a^2 = 0$

$2a^2(a-3) = 0$

$a = 0, a = 3$



Tangent lines:

$\boxed{\begin{array}{l}\text{at } (0,0) \\ y=0\end{array}}$

$\boxed{\begin{array}{l}\text{at } (3,27) \\ m=27 \\ y-27=27(x-3) \\ y=27x-54\end{array}}$